Journal of Statistical Physics, Vol. 34, Nos. 5/6, 1984

Surface Properties of a Classical Two-Dimensional One-Component Plasma: Exact Results

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Received September 30, 1983

At one special temperature, the equilibrium statistical mechanics of a classical two-dimensional one-component plasma can be worked out exactly. This model is used for computing the density profile and the two-body correlation function for three kinds of electrified interfaces: charge particles attracted by a charged plate, charged particles near the permeable boundary of a semiinfinite background, charged particles near the interface between two backgrounds of different densities. Sum rules are discussed.

KEY WORDS: Coulomb systems; plasmas; surface properties; electrified interfaces.

1. INTRODUCTION

The behavior of charged fluids near a surface or an interface is currently attracting much interest. Possible applications are to metallic surfaces, metal–electrolyte interfaces, plasma physics, biophysics, etc. . . . . The simplest model, the one-component plasma, has recently been studied theoretically[1-3] and by numerical simulation.[4]

In two dimensions, there is a soluble model: at one special temperature, the equilibrium statistical mechanics of a classical two-dimensional one-component plasma can be worked out exactly, for both bulk[5,6] and surface properties,[7,8] by using methods from the theory of random matrices.[9,10] This model is a system of identical particles of charge e which interact through the two-dimensional Coulomb potential: the interaction

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energy between two particles at a distance r from one another is

$$e^2 w(r) = -e^2 \ln(r/L)$$

where L is a length scale. Usually, the particles are supposed to be embedded in a uniform background of opposite charge. The dimensionless coupling constant is $\Gamma = e^2/k_B T$, where k_B is Boltzmann's constant and T is the temperature. Exact results were obtained for the special value $\Gamma = 2$.

In the present paper, we consider three more cases in which the structure of the two-dimensional system at an interface can be studied by explicit exact calculations:

(a) A uniformly charged plate (i.e., a line in the two-dimensional model) attracts on one of its sides an atmosphere of charged particles; there is no background.

(b) A uniform background fills a half-space; there is no background in the other half-space. The boundary between these two half-spaces is permeable to a fluid of charged particles.

(c) Two backgrounds with different uniform densities are separated by a plane boundary (i.e., a line in the two-dimensional model). Charged particles adjust their density to this discontinuous background density.

For these three cases, at $\Gamma = 2$, it is possible to compute exactly the density profile of the particles and their two-body density; this is done in Section 2. These quantities obey a number of sum rules, which will be discussed in Section 3.

These exact results in a special case can provide a test bench for approximations to be used in more general situations. This will be the subject of a forthcoming publication.

2. DENSITY PROFILES AND CORRELATIONS

The method of calculation is very similar to the one which has been used for other surface problems.[7,8] We start with a system of circular symmetry. The interface is a circle of radius R. We compute the densities, and afterwards take a limit $R \to \infty$ for obtaining a plane interface. In terms of the total potential energy $V(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N)$ for a system of N particles, the n-body density is

$$\rho^{(n)}(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N) = \frac{N!}{(N-n)!} \frac{\int \exp(-V/k_B T)\, d\mathbf{r}_{n+1} \ldots d\mathbf{r}_N}{\int \exp(-V/k_B T)\, d\mathbf{r}_1 \ldots d\mathbf{r}_N} \tag{2.1}$$

The origin is at the center of the circle, and the position $\mathbf{r}_i$ of the ith particle can be represented either by the polar coordinates (r_i, θ_i) or by the complex number $Z_i = r_i \exp(i\theta_i)$. Up to a constant, the total potential energy is always of the form

$$V=\sum_{i=1}^{N} v(r_i)-e^2\sum_{N\geqslant i>j\geqslant 1}\ln|\mathbf{r}_i-\mathbf{r}_j| \tag{2.2}$$

where $v(r_i)$ is the interaction of the ith particle with the charged interface or with the background and the last term is the interaction between the particles. When $\Gamma=e^2/k_BT=2$, this interaction between the particles contributes to the Boltzmann distribution $\exp(-V/k_BT)$ by a factor

$$\exp\left[(e^2/k_BT)\sum_{i>j}\ln|\mathbf{r}_i-\mathbf{r}_j|\right]=\left|\prod_{i>j}(Z_i-Z_j)\right|^2 \tag{2.3}$$

The product in (2.3) is a Vandermonde determinant which can be expanded as

$$\prod_{i>j}(Z_i-Z_j)=\sum_P \epsilon_P Z_{\alpha_2}Z_{\alpha_3}^2\cdots Z_{\alpha_N}^{N-1} \tag{2.4}$$

where $P=(\alpha_1,\alpha_2,\ldots,\alpha_N)$ is some permutation of $(1,2,\ldots,N)$ and the sum runs on all permutations; ϵ_P is the sign of the permutation. The key point is that the angular integrations which occur in the calculation of the one-body or many-body densities (2.1) can be performed at once, by using (2.4) in (2.3) and the orthogonality property

$$\int_0^{2\pi} Z_i^p Z_i^{*q}\,d\theta_i=2\pi r_i^{2p}\delta_{pq} \tag{2.5}$$

One finds for the one-body density

$$\rho^{(1)}(r)=\exp\left[-(1/k_BT)v(r)\right]K(r^2) \tag{2.6}$$

where

$$K(r^2)=\sum_{l=0}^{N-1}\frac{r^{2l}}{\int\exp\left[-(1/k_BT)v(r)\right]r^{2l}\,d\mathbf{r}} \tag{2.7}$$

and for the truncated two-body density

$$\begin{aligned}\rho_T^{(2)}(\mathbf{r}_1,\mathbf{r}_2)&=\rho^{(2)}(\mathbf{r}_1,\mathbf{r}_2)-\rho^{(1)}(r_1)\rho^{(1)}(r_2)\\&=-\exp\{-(1/k_BT)\left[v(r_1)+v(r_2)\right]\}|K(Z_1Z_2^*)|^2\end{aligned} \tag{2.8}$$

2.1. Uniformly Charged Plate and Charged Particles (Case a)

We start with a circle of radius R uniformly charged with a linear charge density $-e\sigma$. Outside the circle, there are N particles of charge e.

In (2.2),

$$v(r) = 2\pi R\sigma e^2 \ln r \tag{2.9}$$

All integrals in (2.1) and (2.7) converge provided that $N \leqslant 2\pi R\sigma - 1$. Thus, the charged circle of total charge $-2\pi R\sigma e$ can attract at most $2\pi R\sigma - 1$ particles of charge e; if we try to put more particles, they evaporate to infinity. In the following, we choose $N = 2\pi R\sigma - 1$. Thus only one particle is missing for the system to be neutral; in the limit of an infinite system, it will become neutral, since the charge per particle will go to zero.

With $v(r)$ given by (2.9), the integrals [to be taken for r in the range (R, ∞)] and the sum in (2.7) are elementary. One finds from (2.6)

$$\rho^{(1)}(r) = \frac{R^2}{\pi r^4}\left[\frac{1-(R^2/r^2)^{2\pi R\sigma}}{(1-R^2/r^2)^2} - 2\pi R\sigma \frac{(R^2/r^2)^{2\pi R\sigma - 1}}{1-R^2/r^2}\right] \tag{2.10}$$

Finally, the distance x to the charged circle is defined by

$$r = R + x \tag{2.11}$$

and the limit of (2.10) as $R \to \infty$ for fixed values of σ and x is taken. The density profile becomes

$$\rho^{(1)}(x) = \frac{1}{4\pi x^2}\left[1 - (1 + 4\pi\sigma x)\exp(-4\pi\sigma x)\right] \tag{2.12}$$

This profile is plotted in Fig. 1. For large x, $\rho^{(1)}(x)$ has only an

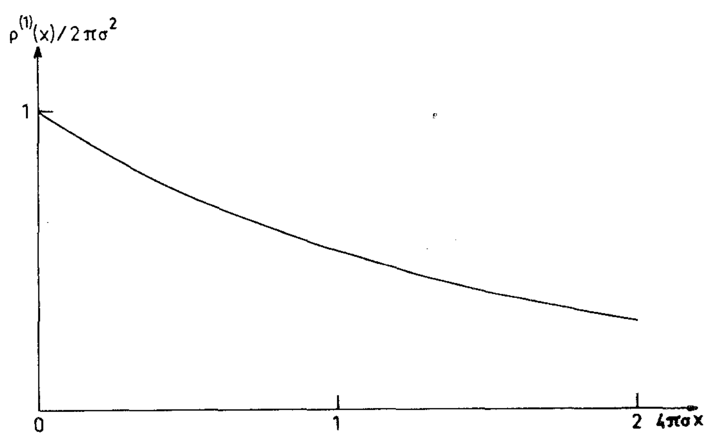

Fig. 1. The density profile $\rho^{(1)}(x)$ for charged particles attracted by a uniformly charged plate; there is no background.

algebraic decay, behaving like $1/4\pi x^2$. The overall neutrality condition

$$-\sigma + \int_0^\infty \rho^{(1)}(x)\,dx = 0 \tag{2.13}$$

is satisfied.

The calculation of the truncated two-body density from (2.8) is very similar. When the limit $R \to \infty$ is taken, it is convenient to choose a new origin on the charged plate, with the x axis normal to the plate and the y axis along the plate; a pair of particles is described by their distances x_1 and x_2 to the plate and the difference y of their coordinates along the plate. One finds

$$\begin{aligned}\rho_T^{(2)}(x_1, x_2; y) &= -\left|\rho^{(1)}\left(\frac{x_1 + x_2 - iy}{2}\right)\right|^2 \\ &= -\frac{1}{\pi^2\left[(x_1 + x_2)^2 + y^2\right]^2} \\ &\quad \times \Big\{1 - 2\big[(1 + 2\pi\sigma(x_1 + x_2))\cos 2\pi\sigma y \\ &\qquad + 2\pi\sigma y \sin 2\pi\sigma y\big]\exp\left[-2\pi\sigma(x_1 + x_2)\right] \\ &\quad + \left[(1 + 2\pi\sigma(x_1 + x_2))^2 + (2\pi\sigma y)^2\right] \\ &\quad \times \exp\left[-4\pi\sigma(x_1 + x_2)\right]\Big\}\end{aligned} \tag{2.14}$$

When $x_1 \to \infty$ or $x_2 \to \infty$, $\rho_T^{(2)}$ has an algebraic decay governed by its term $-1/\pi^2[(x_1 + x_2)^2 + y^2]^2$. When $y \to \infty$, $\rho_T^{(2)}$ has an algebraic decay like $-4\sigma^2 \exp[-4\pi\sigma(x_1 + x_2)]/y^2$.

It is also of interest to consider the integrated quantity

$$s(y) = \int_0^\infty dx_1 \int_0^\infty dx_2\, \rho_T^{(2)}(x_1, x_2; y) \tag{2.15}$$

When $|y| \to \infty$, this quantity decays like $-3/2\pi^2\Gamma y^2$; note that there are contributions to this asymptotic form not only from the term of order $1/y^2$ in $\rho_T^{(2)}$, but also from the term of order $1/[(x_1 + x_2)^2 + y^2]^2$ after it has been integrated upon x_1 and x_2.

2.2. Semiinfinite Background with Permeable Boundary (Case b)

We now start with a disk of radius R filled with a uniform background of charge density $-e\rho$, and N particles of charge e which freely move inside or outside the disk. It is convenient to choose the unit of length in such a way that $\rho = 1/\pi$ (this means that the average interparticle distance a in the bulk, defined by $\rho = 1/\pi a^2$, here is $a = 1$). The background–

particle interaction $v(r)$ is, up to a constant,

$$\begin{aligned} v(r) &= \frac{e^2}{2} r^2, \qquad & r \leqslant R \\ v(r) &= \frac{e^2}{2} R^2 + e^2 R^2 \ln \frac{r}{R}, \qquad & r \geqslant R \end{aligned} \tag{2.16}$$

All integrals in (2.1) and (2.7) converge provided that $N \leqslant R^2 - 1$. This means again that the maximum number $R^2 - 1$ of particles that the system can hold is such that only one particle is missing for the system to be neutral; we do choose $N = R^2 - 1$. The integrals in (2.7) then are

$$\int_0^\infty \exp[-(1/k_B T) v(r)] r^{2l} d\mathbf{r} = \pi \left[\gamma(l+1, R^2) + \frac{R^{2(l+1)}}{R^2 - l - 1} \exp(-R^2) \right] \tag{2.17}$$

where γ is the incomplete gamma function

$$\gamma(l+1, R^2) = \int_0^{R^2} e^{-u} u^l \, du \tag{2.18}$$

In the limit $R \to \infty$, the dominant values of l in (2.7) are close to R^2 and the gamma function (2.18) can be replaced by its asymptotic form[11]

$$\gamma(l+1, R^2) \sim \left(\frac{\pi}{2} \right)^{1/2} R \exp(l \ln l - l) \left[1 + \Phi\left(\frac{R^2 - l}{R\sqrt{2}} \right) \right] \tag{2.19}$$

where Φ is the error function

$$\Phi(t) = \frac{2}{\sqrt{\pi}} \int_0^t e^{-u^2} du$$

The summation (2.7) can be replaced by an integral upon $t = (R^2 - l)/R\sqrt{2}$. Defining again the coordinate x normal to the interface by (2.11), one finds, in the limit $R \to \infty$, for a given value of x,

$$\rho^{(1)}(x) = f(x) \frac{2}{\pi^{3/2}} \int_0^\infty \frac{\exp[-2tx\sqrt{2} - t^2]}{1 + \Phi(t) + \exp(-t^2)/t\sqrt{\pi}} dt \tag{2.20}$$

where

$$\begin{aligned} f(x) &= \exp(-2x^2), \qquad & x < 0 \\ f(x) &= 1, \qquad & x > 0 \end{aligned} \tag{2.21}$$

The integral representation (2.20) of the density profile has been numerically computed; this profile is plotted in Fig. 2. The density at the interface is $\rho^{(1)}(0) = 0.36022\rho$. For $x \to +\infty$, $\rho^{(1)}(x)$ again behaves like $1/4\pi x^2$. For $x \to -\infty$, $\rho^{(1)}(x)$ approaches ρ in a way which is essentially

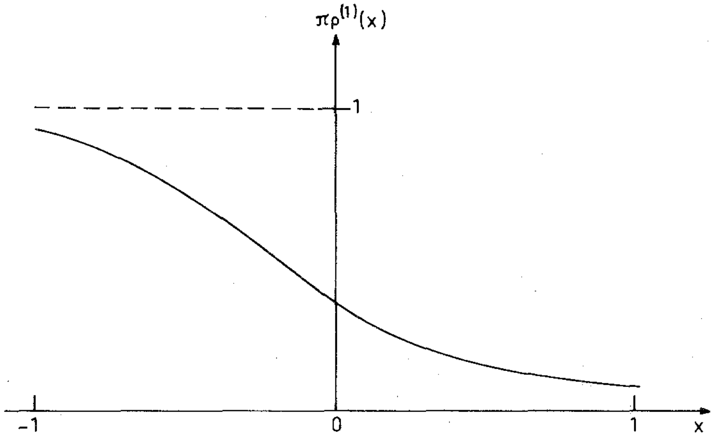

Fig. 2. The density profile $\rho^{(1)}(x)$ for charged particles in a semiinfinite background with a permeable boundary; the background occupies the region $x < 0$.

Gaussian:

$$\rho^{(1)}(x) \sim \rho\left[1 - \frac{\exp(-x^2)}{2|x|^3\pi^{1/2}}\right] \tag{2.22}$$

One can verify after some manipulations that the overall neutrality condition

$$\int_{-\infty}^{0}\left[\rho^{(1)}(x) - \rho\right]dx + \int_{0}^{\infty}\rho^{(1)}(x)\,dx = 0 \tag{2.23}$$

is satisfied.

The truncated two-body density for a plane interface is found to be

$$\rho_T^{(2)} = -f(x_1)f(x_2)\left|\frac{2}{\pi^{3/2}}\int_0^{\infty}\frac{\exp\left[-t(x_1 + x_2 - iy)\sqrt{2} - t^2\right]}{1 + \Phi(t) + \exp(-t^2)/t\sqrt{\pi}}\,dt\right|^2 \tag{2.24}$$

The asymptotic properties of (2.24) when $|x_1 + x_2 - iy| \to \infty$ can be obtained by integration by parts:

$$\rho_T^{(2)}(x_1, x_2; y) \sim -\frac{f(x_1)f(x_2)}{\pi^2\left[(x_1 + x_2)^2 + y^2\right]^2} \tag{2.25}$$

Therefore, as particle 2 recedes in a direction such that $x_2 \to -\infty$, $\rho_T^{(2)}$ decays in a way which is essentially Gaussian; If particle 2 recedes in a direction such as $x_2 \to +\infty$, $\rho_T^{(2)}$ decays algebraically as the inverse fourth power of the distance. Finally, along the interface, i.e., if $|y| \to \infty$ for fixed values of x_1 and x_2, $\rho_T^{(2)}$ also decays as the inverse fourth power $1/y^4$.

The integrated quantity

$$s(y) = \int_{-\infty}^{\infty} dx_1 \int_{-\infty}^{\infty} dx_2 \, \rho_T^{(2)}(x_1, x_2; y) \tag{2.26}$$

has an asymptotic form which is obtained from the contributions $x_1, x_2 \geqslant 0$ to (2.25); it behaves like $-1/\Gamma\pi^2 y^2$ as $|y| \to \infty$.

2.3. Two Backgrounds with Different Densities (Case c)

We start with a disk of radius R filled with a background of charge density $-\alpha_1 e/\pi$; outside the disk, there is a background of charge density $-\alpha_2 e/\pi$, which extends to infinity. We introduce at once an infinite number of particles of charge e. The background–particle interaction $v(r)$ is, up to a constant,

$$\begin{aligned} v(r) &= \frac{e^2}{2}\alpha_1 r^2, \qquad r \leqslant R \\ v(r) &= \frac{e^2}{2}(\alpha_1 - \alpha_2)R^2 + \frac{e^2}{2}\alpha_2 r^2 + e^2(\alpha_1 - \alpha_2)R^2 \ln\frac{r}{R}, \qquad r \geqslant R \end{aligned} \tag{2.27}$$

The integrals in (2.7) are

$$\int_0^{\infty} \exp\left[-(1/k_B T)v(r)\right] r^{2l} d\mathbf{r} = \pi\left[\frac{\gamma(l+1, \alpha_1 R^2)}{\alpha_1^{l+1}} + \frac{\exp\left[(\alpha_2 - \alpha_1)R^2\right]\Gamma\left(l + 1 + (\alpha_2 - \alpha_1)R^2, \alpha_2 R^2\right)}{\left(\alpha_2 R^2\right)^{(\alpha_2 - \alpha_1)R^2} \alpha_2^{l+1}}\right] \tag{2.28}$$

where

$$\Gamma(n+1, z) = \int_z^{\infty} e^{-u} u^n \, du \tag{2.29}$$

The summation in (2.7) now runs for l from 0 to ∞. Using again the asymptotic form (2.19), the related asymptotic form

$$\Gamma(n+1, z) = \left(\frac{\pi}{2}\right)^{1/2} \sqrt{z}\, \exp(n \ln n - n)\left[1 - \Phi\left(\frac{z - n}{\sqrt{2z}}\right)\right] \tag{2.30}$$

and replacing the summation in (2.7) by an integration upon $t = (\alpha_1 R^2 - l)/R\sqrt{2}$, one now finds[2] in the limit $R \to \infty$,

$$\rho^{(1)}(x) = g(x)\frac{2}{\pi^{3/2}}\int_{-\infty}^{\infty} \frac{\exp(-2tx\sqrt{2})}{\frac{1}{\sqrt{\alpha_1}}\exp\left(\frac{t^2}{\alpha_1}\right)\left[1+\Phi\left(\frac{t}{\sqrt{\alpha_1}}\right)\right] + \frac{1}{\sqrt{\alpha_2}}\exp\left(\frac{t^2}{\alpha_2}\right)\left[1-\Phi\left(\frac{t}{\sqrt{\alpha_2}}\right)\right]}\,dt \tag{2.31}$$

where

$$\begin{aligned} g(x) &= \exp(-2\alpha_1 x^2), \qquad x \leqslant 0 \\ g(x) &= \exp(-2\alpha_2 x^2), \qquad x \geqslant 0 \end{aligned} \tag{2.32}$$

It can be checked that (2.31) reduces to (2.20) in the case $\alpha_1 = 1$ $\alpha_2 = 0$.

The integral representation (2.31) of the density profile has been numerically computed in the case $\alpha_1 = 1$, $\alpha_2 = 0.5$; this profile is plotted in Fig. 3. Now, $\rho^{(1)}(x)$ approaches the background density in a way which is

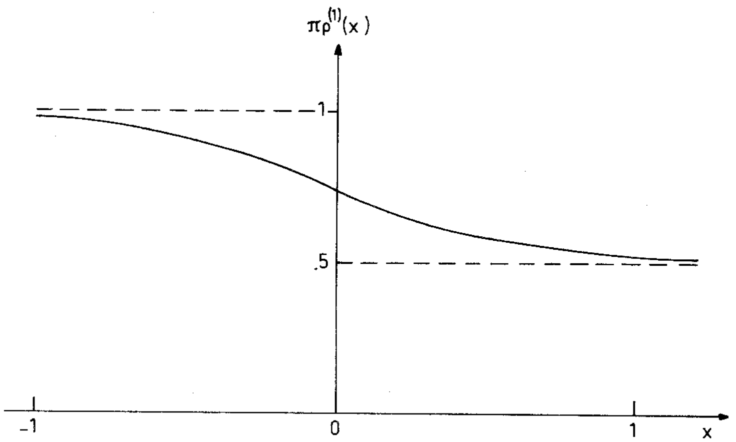

Fig. 3. The density profile $\rho^{(1)}(x)$ for charged particles near the plane interface between two backgrounds of different densities; the background density is $1/\pi$ for $x < 0$ and $0.5/\pi$ for $x > 0$.

[2] This result has been independently obtained by L. Blum (to be published).

essentially Gaussian on both sides of the interface. The overall neutrality condition is again satisfied:

$$\int_{-\infty}^{0}\left[\rho^{(1)}(x)-\frac{\alpha_1}{\pi}\right]dx+\int_{0}^{\infty}\left[\rho^{(1)}(x)-\frac{\alpha_2}{\pi}\right]dx=0 \tag{2.33}$$

The truncated two-body density for a plane interface is found to be

$$\rho_T^{(2)}(x_1,x_2;y) = -g(x_1)\,g(x_2)\left|\frac{2}{\pi^{3/2}}\int_{-\infty}^{\infty}\frac{\exp\left[-t(x_1+x_2-iy)\sqrt{2}\right]}{\frac{1}{\sqrt{\alpha_1}}\exp\left(\frac{t^2}{\alpha_1}\right)\left[1+\Phi\left(\frac{t}{\sqrt{\alpha_1}}\right)\right]+\frac{1}{\sqrt{\alpha_2}}\exp\left(\frac{t^2}{\alpha_2}\right)\left[1-\Phi\left(\frac{t}{\sqrt{\alpha_2}}\right)\right]}\,dt\right|^2 \tag{2.34}$$

Since the integration on t in (2.34) now runs from $-\infty$ to $+\infty$, as $|x_1+x_2-iy|\to\infty$, $\rho_T^{(2)}$ decreases faster than any inverse power of this quantity: the correlations decay faster than any inverse power law in any direction. In the direction parallel to the interface, $\rho_T^{(2)}$ will decay with exponentially damped oscillations as a function of y.

3. SUM RULES

The exact solutions which have been obtained in Section 2 are expected to satisfy a number of sum rules which will now be discussed.

3.1. Overall Electroneutrality

The total charge in the interfacial layer is expected to vanish. This has indeed been already verified for the three cases considered here, in (2.13), (2.23), (2.33).

3.2. Potential

Let $\phi(x)$ be the electrostatic potential. We choose $\phi(-\infty)=0$. Let $\rho_Q(x)$ be the difference between the particle density $\rho^{(1)}(x)$ and the background density at x.

We consider first case (c) (two backgrounds of different densities

$\rho_1 = \alpha_1/\pi$ for $x < 0$ and $\rho_2 = \alpha_2/\pi$ for $x > 0$). The potential at the interface is

$$\phi(0) = 2\pi e \int_{-\infty}^{0} \rho_Q(x) x \, dx \tag{3.1}$$

and the total potential difference across the interfacial layer is

$$\phi(\infty) = 2\pi e \int_{-\infty}^{\infty} \rho_Q(x) x \, dx \tag{3.2}$$

The potentials are expected to obey two sum rules[1-3]:

$$e\phi(\infty) + \mu(\rho_2) = \mu(\rho_1) \tag{3.3}$$

where $\mu(\rho)$ is the chemical potential in the bulk phase of density ρ, and

$$p(\rho_1) - p(\rho_2) = (\rho_1 - \rho_2) e\phi(0) + \rho_2 e\phi(\infty) \tag{3.4}$$

where $p(\rho)$ is the pressure in the bulk phase of density ρ. For a two-dimensional one-component plasma, the equation of state has the simple form[12]

$$p = \rho(k_B T - e^2/4) \tag{3.5}$$

and therefore the chemical potential difference is

$$\mu(\rho_2) - \mu(\rho_1) = \left(k_B T - \frac{e^2}{4}\right) \ln \frac{\rho_2}{\rho_1} \tag{3.6}$$

Here, $k_B T = e^2/2$. Using (2.31) in (3.1) and (3.2), one can show after some manipulations[13] that the sum rules (3.3) and (3.4) are indeed satisfied.

In case (b) (semiinfinite background with a permeable boundary), the total potential difference (3.2) and $\mu(\rho_2)$ both diverge ($\rho_2 = 0$). The sum rule (3.3) is still satisfied, in the sense that these divergences are consistent with one another and with the asymptotic form $1/4\pi x^2$ for $\rho^{(1)}(x)$. The sum rule (3.4) takes the simpler form

$$p = \rho e \phi(0) \tag{3.7}$$

where p and ρ are the bulk pressure and density. Using (2.20) in (3.1), one can show that (3.7) is indeed satisfied.

3.3. Correlations

The truncated two-body density $\rho_T^{(2)}$ satisfies sum rules,[14,15] the validity of which is related to the asymptotic behavior of $\rho_T^{(2)}$.

For case (a), the asymptotic behavior of $\rho_T^{(2)}$ and of its integral $s(y)$ defined by (2.15) have been described in Section 2.1. The y^{-2} behavior is the same as for a two-dimensional one-component plasma (with a background) near a plane hard wall. Note, however, that the linear response argument which has been used elsewhere[7,8] for predicting the coefficient in front of y^{-2} in the asymptotic form of $s(y)$ is *not* applicable here, because the decay of $\rho_T^{(2)}(x_1, x_2; y)$ in the x direction (i.e., when x_1 or $x_2 \to \infty$) is not fast enough. The perfect-screening sum rule[14]

$$\int_0^\infty dx_2 \int_{-\infty}^\infty dy\, \rho_T^{(2)}(x_1, x_2; y) = -\rho^{(1)}(x_1) \tag{3.8}$$

is satisfied. The dipole moment of $\rho_T^{(2)}$ does not vanish, because the decay of $\rho_T^{(2)}$ is not fast enough; instead, it obeys the sum rule[15]

$$-2\pi\Gamma \int_0^\infty dx_2 \int_{-\infty}^\infty dy\, (x_2 - x_1)\rho_T^{(2)}(x_1, x_2; y) = \frac{\partial \rho^{(1)}(x_1)}{\partial \sigma} \tag{3.9}$$

For case (b), the asymptotic behavior of $\rho_T^{(2)}$ and of its integral $s(y)$ defined by (2.26) have been described in Section 2.2. Again, the coefficient in front of y^{-2} in the asymptotic form of $s(y)$ cannot be predicted by the general theory of Ref. 7. Since $\rho_T^{(2)}$ decays as an inverse fourth power, both the monopole and dipole moment sum rules of Ref. 14 are satisfied:

$$\int_{-\infty}^\infty dx_2 \int_{-\infty}^\infty dy\, \rho_T^{(2)}(x_1, x_2; y) = -\rho^{(1)}(x_1) \tag{3.10}$$

$$\int_{-\infty}^\infty dx_2 \int_{-\infty}^\infty dy\, (x_2 - x_1)\rho_T^{(2)}(x_1, x_2; y) = 0 \tag{3.11}$$

Higher-order multipole moments of $\rho_T^{(2)}$ are not defined.

For case (c), as described in Section 2.3, $\rho_T^{(2)}$ decays faster than any inverse power law. Correspondingly, the sum rules (3.10) and (3.11) are valid, and more generally we expect *all* higher-order electrical moments of $\rho_T^{(2)}$ to vanish. A fast decay of $\rho_T^{(2)}$ along the interface had already been obtained for a charged fluid near a perfectly conducting wall, in weak-coupling theories[17,18]; such a decay, faster than any inverse power law, seems to be a general feature along a plane interface between two conducting media.

ACKNOWLEDGMENT

The author is indebted to A. Alastuey for his help with some numerical calculations.

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